Developing Mathematical Literacy in Welsh Secondary Schools

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ABSTRACT

Standards of literacy and numeracy in Wales are a cause for concern. Twenty per cent of 16- to 19-year-olds have literacy levels at or below entry level; of greater concern, 60 per cent are estimated to have standards of numeracy at or below that level (WG, 2011).

In the 2009 cycle of the Programme for International Student Assessment (PISA) Wales performed significantly lower than the Organization for Economic Cooperation and Development average in both reading and mathematical literacy and significantly lower than the other countries in the United Kingdom (Bradshaw et al., 2010).

The minister's keynote address 'Raising Schools Standards', delivered on 29 June 2011 to the Institute of Welsh Affairs, announced the intention to introduce a new National Literacy and Numeracy Framework, including national tests of literacy and numeracy (Andrews, 2011b).

In July 2011 an action research project was established at Swansea Metropolitan University, in partnership with six secondary schools in five local authorities in Wales, to investigate the development of reading and mathematical literacies across the curriculum. This paper reports on the development of mathematical literacy.

Mathematical literacy under PISA is defined as 'an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen' (Bradshaw et al., 2010).

Teachers were co-researchers in action research over the 2011/12 academic year focusing on Year 9. During the year, university researchers

joined with teachers in network meetings to discuss the development of intervention strategies. A socio-constructivist view was taken of the learning of mathematics. University researchers also observed lessons and interviewed teachers about their interventions. Alongside this qualitative analysis, the project incorporated pre- and post-testing of all pupils using PISA-style assessments to inform a post hoc analysis of the efficacy of different strategies.

Background

Concerns about standards of numeracy in our schools are not new. In the last quarter of the twentieth century, concerns over the standards achieved in key skills were raised by the 'Great Debate' initiated by James Callaghan in his Ruskin College speech of 1976. Such concerns have remained at the heart of UK education policy ever since, through the various iterations of the National Curriculum and its assessment systems, the National Strategy guidance in England (Jones and Tanner, 2008), and, more recently in Wales, the National Literacy and Numeracy Framework (WG, 2013). Standards are extremely difficult to monitor over time as systems change and the nature of the concepts and skills under examination drifts.

Numeracy or Mathematical Literacy

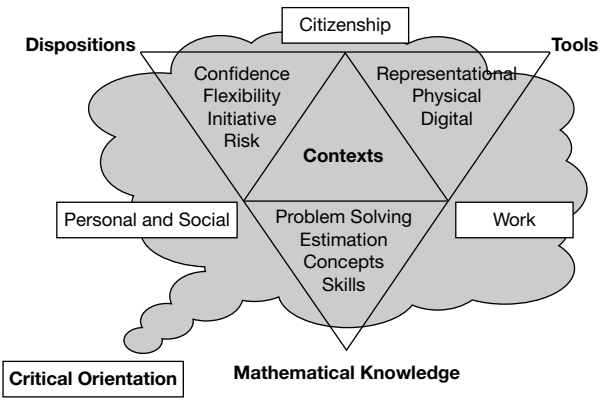
There is no universally accepted definition of the term numeracy, which would allow us to distinguish it from mathematics in general (WGSR, 2012). Although the term 'numeracy' is used in most education systems in the English-speaking world, its meaning ranges from the recall of simple facts and the mechanical processing of simple arithmetical algorithms, to a sophisticated use of mathematics in the modelling of real world problems. Cockcroft (1982) described 'being numerate' as possessing an 'at-homeness' with numbers and an ability to use mathematical skills to cope confidently with the practical demands of everyday life (Jones and Tanner, 2008; Goos et al., 2010). In the USA, the term 'quantitative literacy' is more common (Steen, 2001), which is a far wider and more encompassing construct that includes confidence, the history and place of mathematics in society, logical thinking and decision making and problem solving within a wide range of practical and real life contexts.

In our research, we have always taken a broader view of numeracy, demanding that a numerate person has not only a secure knowledge of numerical facts and processes but also the capability and disposition to construct personal approaches to the solution of problems, which are based on self-knowledge of individual strengths and weaknesses (Jones and Tanner, 2008). To be numerate is to be able to mathematize situations, in a range of contexts, from the real world or the mathematical world to solve problems. The difficulty of the problems that a numerate person might be expected to solve will depend on the extent of their mathematical education. The numerate person has mathematical knowledge, which is accessible for problem solving, and the confidence to use it creatively (Tanner and Jones, 2000, 2002; Jones and Tanner, 2008).

We find the model offered in Figure 1 (Goos et al., 2010) helpful in defining the complexity and scope of the domain under consideration.

A numerate person requires *mathematical knowledge*, conceptual understanding and problem-solving skills, *positive dispositions* such as confidence and flexibility of mind and the ability to use appropriate *mathematical tools*, which might be representational, physical or digital. These attributes must be accesible in a range of different contexts and taken together provide a critical orientation to numeracy that allows citizens to mathematics, evaluate and interpret quantitative, algebraic,

Figure 1 A model for numeracy in the twenty-first century



Source: Goos, Geiger and Dole (2010)

spacial or probabilistic information appropriately in complex situations (Goos et al., 2010).

The current concerns about standards of numeracy in Wales have placed great emphasis on the poor performance of Wales in comparison with England, Scotland and Northern Ireland in the recent Programme of International Student Assessment (PISA) (Bradshaw et al., 2010) (Table 1).

Table 1: Home nation PISA scores for mathematical literacy

	2006	*2009*
OECD average	498	496
United Kingdom	495	492
Scotland	506	499
England	495	493
Northern Ireland	494	492
Wales	484	472

Source: Bradshaw et al. (2010)

PISA scores are standardized to have a mean of 500 and a standard deviation of 100. The scores in mathematical literacy in Wales are significantly worse than the OECD and UK averages and the direction of travel is negative (Bradshaw et al., 2010). It is interesting to note that in the UK and Wales, the trend was downwards at a time when GCSE results were continuing to improve.

We do not wish to read too much into such trends, as the samples used for PISA standardization have not been stable over the period and such an analysis would not be statistically valid. Performance in Trends in International Mathematics and Science Study (TIMSS) for England has improved while much political capital has been made of an apparent deterioration in performance in PISA (Ruthven, 2011a, 2011b). Different constructs of mathematical attainment are being assessed. Furthermore, there are socio-economoic indicators which, although making no claim to causality, suggest that a comparison of pupil performance in England and Wales might not be completely fair (Gorad et al., 2004).

However, the poor performance of Wales in the PISA study created a climate in which dealing with poor standards of literacy and numeracy became a political imperative with a target that 'We should aim to be in the top 20 of school systems measured in the PISA scores in 2015' (Andrews,

2011a). However, although the constructs assessed by PISA, TIMSS and GCSE have some overlap, they are far from congruent. The Andrews (2011a) 20-point plan demanded that PISA- style assessments be introduced for 15-year-old pupils and that pupils in Years 8 and 9 be prepared for them.

In PISA, the term *mathematical literacy*

> emphasises mathematical knowledge put to functional use in a multitude of different situations in varied, reflective and insight-based ways … mathematical literacy presupposes, but cannot be reduced to, knowledge of mathematical terminology, facts and procedures … It involves the creative combination of these elements in response to the demands imposed by external situations. (OECD, 2010: 84)

In PISA assessments, students are typically required to extrapolate from what they have learned in school, to apply mathematical knowledge to authentic problems in a variety of contexts (OECD, 2010). Levels of general knowledge are assumed that are unlikely to be consistent over impoverished backgrounds.

The contexts sometimes make high demands of literacy and many are presented with no obvious mathematical structure – this must be imposed by the student. This is very different from the style of GCSE question which was common in Wales in 2009. PISA aims to assess the student's ability to pose, formulate, solve and interpret problems using mathematics (OECD, 2010). The student must have a degree of resilience to persist in the face of initial adversity. In this project we explored the use of PISA-style problem situations to teach and assess students' numeracy.

Teaching Mathematical Literacy

The lack of consensus about the definition of numeracy or quantitative literacy makes it difficult to ensure that the same constructs are being considered when reviewing the literature on effective pedagogy. Although data drawn from international studies such as TIMSS and PISA are often used to justify particular pedagogical practices, a comparison of high-attaining systems more often than not reveals a wide variation in practices that are sometimes diametrically opposed to each other, suggesting that 'good practice is a culturally determined entity' (Clark et al. (2006) in State of Victoria Department of Education and Early Childhood

Development, 2009: 18). However, when the wider definition of mathematical literacy described above is considered a number of common features of effective pedagogy appear in the research literacture:

- Sophisticated and flexible scaffolding of concept development (for example, Bauersfeld, 1988; Wood, 1994; Tanner and Jones, 2000; Jones and Tanner, 2002; Jones, 2008; State of Victoria Department of Education and Early Childhood Development, 2009);
- Pupil autonomy and control of strategic thinking, valuing pupils' strategies (for example, Tanner and Jones, 2000, Tanner et al, 2005; Jones and Tanner, 2008; Slavin et al., 2009; State of Victoria Department of Education and Early Childhood Development, 2009);
- Connectionist or constuctivist orientation (for example, Steen, 2001; Tanner et al, 2002; Askew and Brown, 2003; Jones, 2008; State of Victoria Department of Education and Early Childhood Development, 2009);
- Dialogic and discourse-based teaching (for example, Tanner and Jones, 2000; Jones and Tanner, 2002, 2008; Alexander, 2004; Mercer and Sams, 2006; Jones, 2008; State of Victoria Department of Education and Early Childhood Development, 2009);
- Collaborative and group approaches such as Complex Instruction, using challenging tasks and groupworthy problems in a wide range of contexts (for example, Cohen, 1986; Cohen et al., 1999; Tanner and Jones, 2000; Tanner et al, 2005; Steen, 2001; Tanner et al., 2002; Boaler, 2006; Slavin et al., 2009; Sebba et al., 2011; Goos et al., 2010; WGSR, 2012).

Many of these terms are loosely defined in the literature. In previous research, we have analysed how the term 'scaffolding' can be used to describe a range of contingent teaching strategies ranging from rigid, pre-planned, funnelling questioning through more flexible forms to a more fully elaborated reflective discourse, as the degree of pupil autonomy and pupil control of strategic thinking and learning process increases. Teachers using more flexible and dialogical forms of scaffolding perform better than those using more rigid forms of instruction (Tanner and Jones, 2000; Tanner et al., 2005).

In an action research project into the teaching of numeracy, a quasi-experiment, using control and intervention classes, compared the performance of 450 pupils, aged 11 to 13, using pre-, post- and delayed written and mental numeracy tests (Jones, 2008; Jones and Tanner, 2008). Teachers were classified as Reflective Inquirers' or 'Performers'. Within

the Reflective Inquirers' classrooms, the pupils had a high degree of autonomy and responsibility for their own learning. They were expected to contribute actively towards the co-construction of mathematical knowledge, scaffolded flexibly by the social discourse of the lesson. The Performers, by contrast, had tightly structured and planned lessons with clear objectives. Their lessons involved a high level of interaction and pace but were based more on a positive transmission model of teaching than dialogical co-construction of knowledge. The Reflective Inquirers' classes were significantly better than their controls in the tests, with a medium size of effect ($p < .001$, partial eta squared = .06). The Performers showed no discernible advantage over their controls (Jones, 2008; Jones and Tanner, 2008). We have found it helpful to use a musical analogy to illustrate the distinction between the styles of interaction. In classical orchestration, the score is fixed, although open to a degree of interpretation by the Performers. However, the Reflective Inquirers often encouraged pupils to express their own ideas for public discussion and contingent response. 'This style of orchestration is more characteristic of jazz in which the musician's unplanned improvisations in response to stimuli from other players mirrors the teacher's ability to respond in the moment to spontaneous ideas from pupils who have taken the lead' (Tanner et al., 2010: 549).

Strong positive outcomes are often reported for approaches involving collaboration and groupwork (Tanner and Jones, 2000, 2002; Slavin et al., 2009; WGSR, 2012). However, there is no consensus in the literature about the nature of groupwork and the roles to be played by pupils within effective groups. Complex Instruction (CI) aims to achieve equity in heterogeneous classroom situations through specific groupwork strategies. It has three main components: open-ended tasks that require pupils to work interdependently to solve problems; cooperative norms; and specified roles within groups to ensure that all pupils are perceived to have an active part to play (Cohen, 1986; Cohen et al., 1999; Boaler, 2006). Boaler (2006) reports impressive gains in 'Railside School' in the USA compared with traditional approaches. However, when CI was evaluated in a small study in England involving six schools using a version of CI and four control schools, no significant advantage was found for the CI schools. However, this may be because of the way groupwork was conceived by the teachers and the 'groupworthiness' of the tasks (Sebba et al., 2011; WGSR, 2012). For groupwork to be effective, there should be sufficient problematic in the task to demand interdependent working (Tanner and Jones, 2000, 2002).

Setting problems from contexts from across the curriculum and from the real world encourages pupils to construct connected knowledge and have positive outcomes for motivation and understanding (Steen, 2001; Tanner et al., 2002). Real world contexts are often open-ended and more difficult to mathematize, so often include sufficient problematic to be groupworthy (Tanner and Jones, 2002).

Exactly how such pedagogical strategies might be brought into operation effectively in a particular classroom environment is far from clear, particularly in schools where the dominant tradition is transmissive rather than socio-constructivist and dialogical. This is likely to prove to be a challenge for the many teachers who may not have taught or been taught in that way themselves (Tanner and Jones, 2000; Jones and Tanner 2002; Jones, 2008).

Methodology

Because of the complexity of the issues under consideration, the project was conceived as action research. The project was based on a professional learning community established in partnership with six secondary schools across south Wales. The primary aim of this group was to participate, as co-researchers, in an action research project to evaluate and inform whole-school approaches to developing thinking skills in literacy and numeracy. The project lasted for the duration of the 2011/12 academic year and was bookended by pre- and post-testing of that year's Year 9 pupils using PISA-style tests.

The participating schools were approached in July 2011 based on criteria relating to their position in their family groupings and in the value added tables to be in the mid-range for Wales. Six headteachers from five local authorities (LAs) attended an initial meeting to establish the aims of the project and the ethical procedures that would be followed. These included guarantees of anonymity for schools and teachers. It was agreed that any teachers observed would give free informed consent and that any data collected during lesson observations would remain confidential between the teacher and the observer. The relationship between university and school staff was to be established as co-researchers.

At the start of the autumn term, one school dropped out due to significant staff illness in key areas leaving five schools from five LAs that completed the project. The project began with members of the university team discussing the project at whole-school staff meetings.

A research network group was established. Each school agreed to send two representatives to each network group meeting – one literacy coordinator and one numeracy coordinator. There were six network meetings spread over the year, interspersed with five lesson observation visits each in literacy and numeracy. This paper focuses on the numeracy aspects of the project.

Lesson observation visits included a preliminary discussion with the teacher about the aims of the lesson, the strategies to be employed and the role of the observer. After the lesson, the observer and teacher acted as co-researchers analysing the teaching and learning that had occurred. Handwritten notes were taken by the observer during visits and were subject to teacher validation.

Network meetings took place in each of the schools and at the university. At these meetings discussions initially took place as a whole group but then split into literacy and numeracy subgroups. Teachers reported back on progress in school since the last meeting and planned the next cycle of action research. Discussions were recorded using hand written notes by a member of the team and validated by the group.

Pre-test questions were taken from the specimen questions published by PISA. Questions were selected to be within range of Year 9 pupils (PISA was targeted at 16-year-old pupils). The questions had PISA rankings varying between 406 and 723. The item classifications were five reproduction, four connection and three reflection. The items made significant reading demands and the imposition of structure. The post-test was structured in the same way and used items that were mathematically equivalent but appeared to be different.

The Strategies Developed in the Action Research

Initial interventions focussed on the reading demands of questions and the development of resilience. The general opinion was that most questions used in mathematics lessons used as few words as possible due to the low literacy skills of some pupils and a desire to focus on mathematics rather than language. Strategies attempted included 'think-pair-share' (TPS) and paired reading, in which pupils read alternate sentences to each other in pairs. Coloured highlighters were used to identify to indicate key information and the question asked. Initially, children found this selection difficult with many slowly highlighting every word in a question. They had to be

taught to slow down this problem entry phase by reading the text two or three times before highlighting key phrases. Wordy questions challenged the expectations of some pupils who initially reacted badly and refused to engage with the text.

One school shared an approach taken from its trilingual policy to help pupils to deal with text that included unfamiliar words by seeding word questions with Welsh or French words and discussing strategies for dealing with unfamiliar words, for example:

> DAEARGRYNFEYDD: A documentary was broadcast about daeargrynfeydd and how often daeargrynfeydd occur. A daearegwr stated: 'In the next 20 years, the chance that a daeargryn will occur in Zed city is two out of three . . .'

Other strategies included working with a shared-class problem using groupwork and TPS. Some teachers experimented with aspects of Complex Instruction by creating rich groupworthy tasks and attempting to establish collaborative norms and specific roles in groups to ensure that pupils were responsible for each other's learning. For example, in one class only one pupil in each group was allowed to write, one to calculate and one to ask questions in the initial phase. After a period working on the problem talk was allowed and groups were told that only one pupil (not yet designated) would be allowed to feed back for the group so all had to understand what to say.

Some teachers attempted developing resilience by slowing down the problem entry phase, making time to explore the question to develop the problem space before making plans. For example, in one class the teacher issued a sheet with the problem context but no questions and used TPS with groups of pupils listing what they noticed about the context and then what questions they might ask. After a brainstorm, the actual questions were released and answered.

The themes underpinning the interventions were largely targeted at:

- dialogical and constructivist approaches;
- the development of resilience;
- the development of a collaborative classroom culture;
- the development of a Community of Inquiry;
- and the development of periods of collective reflection.

However, implementing complex changes to teaching style is far from easy. Lesson observations and network meeting reports indicated that some teachers were reluctant to engage with inquiry or discourse-based

Table 2 Raw mean scores for pre- and post-tests

School	*Pre-test*	*Post-test*
A	4.6	4.9
B	3.8	4.3
C	3.8	4.4
D	2.3	4.6
E	2.8	3.8

strategies. Some teachers had an Instructivist model of teaching which was transmission and practice based. They often presented a problem, explained how they had solved it and set several more near-identical problems with only slightly changed numbers. They justified the approach on the basis that 'These kids need a familiar routine to follow. They can't cope with open-ended work.'

The results of the test showed that all the schools had made some improvement over the year (Table 2).

This is not unexpected after a year of teaching that had included the deliberate targeting of thinking skills associated with mathematical literacy. A post hoc analysis was then conducted based on the qualitative data collected during lesson observations and the reports that the teachers had made at the network meetings.

Post Hoc Analysis of the Quantitative Data

Schools were classified according to whether they engaged with significant elements of discourse- and inquiry-based strategies or maintained a more traditional Instructivist pedagogy. The performance of these two groups of schools was then compared using an analysis of covariance (ANCOVA).

In this analysis, the pre-test was taken as the covariate to adjust for any differences in performance that had existed in the pre-test at the start of the year. The dependent variable was the post-test. The fixed factor was the teaching style Discourse/Inquiry based versus Traditional Instructivist (Table 3).

This advantage to the Discourse/Inquiry-based classes is significant beyond the 0.1% level, with a medium size of effect ($F=22$, $p < 0.001$, partial eta squared = 0.034).

Table 3 Covariate analysis

Dependent variable = Post-test, Covariate = Pre-test, Fixed factor = Teaching Style

Dominant Pedagogy	*n*	*Adjusted Mean*	*Standard Deviation*
Discourse/Inquiry based	306	4.9	0.13
Traditional Instructivist	327	4.1	0.13

These results support the use of the Discourse- and Inquiry-based strategies recommended in the literature, but they also confirm how difficult it is to move teachers on from traditional, Instructivist practices.

Some of the comments made at the final network meeting are revealing:

'We were spoonfeeding them too much, but how can we do anything else?'

This was a comment made by a teacher whom I had observed following a traditional Instructivist lesson structure based on example and practice, a structure that had served them well for the form of GCSE in existence at that time.

Other teachers had recognized the need to make their pupils less dependent and more resilient in the face of challenge:

'We have been spoonfeeding them too much. We are all trying to add more challenge.'

This was a comment made by a teacher who was beginning to develop strategies to give more responsibility for learning to his pupils.

Others recognized the restricted life experiences of many pupils and the difficulty that this posed when asking them to link their mathematical and real world knowledge:

'Our kids don't have the necessary background knowledge and skills to do this sort of work. We have to scaffold them.'

This teacher had focussed on problem entry and the need to understand the context before beginning to plan a solution. They had helped their pupils to develop strategies to help them to be resilient and remain engaged when dealing with problem contexts that included aspects that they did not understand immediately.

Some teachers were very enthusiastic about the changes that they were making in their own teaching style and that of others in their school:

'This lets us focus on what is important – not just passing exams.'

Conclusions

Some teachers in the project schools have re-evaluated their role as a teacher; other teachers remain committed to a more traditional Instructivist pedagogy. In the lessons that we observed, teachers were trying not only to modify to their own practices, but also to modify the expectations of their pupils. In schools that had focussed on attaining grades in a predictable GCSE that made few language demands and provided structure within the questions, the style of question used in PISA appeared quite alien and challenging. We observed negative emotional reactions from pupils in such schools when presented with unexpectedly wordy and complex questions.

In all the project schools, literacy, oracy and problem solving were beginning to play a more prominent role in mathematics lessons.

In all the project schools, lessons had begun to include richer and more challenging tasks. However, some teachers remained uncomfortable about using groupwork in problem solving.

In some schools, more initiative and responsibility was expected from the pupils, but in other schools traditional, Instructivist approaches left pupils in a position of dependency.

Changing expectations of teaching and learning is complex and takes time. Teachers are often driven by the nature of the external examinations for which they prepare their pupils. Sadly, the National Numeracy Tests that are being taken as this paper goes to press are limited in the first year to procedural questions taken out of context. Preparation for these tests may well reward traditional Instructivist approaches. It remains to be seen whether this approach will improve standards of numeracy in Wales.

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